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INTEGRATION OF TWO DIFFERENTIAL FORMS.

BY FERDINAND SHACK, ESQ., NEW YORK CITY.

To integrate $x^m \sin x dx$ and $x^m \cos x dx$. Because $fudv = uv - fvdv$; let $u = \cos x$ and $dv = x^n dx$, then

$$\int x^n \cos x dx = \frac{1}{n+1} x^{n+1} \cos x + \frac{1}{n+1} \int x^{n+1} \sin x dx;$$

$$\therefore \int x^{n+1} \sin x dx = -x^{n+1} \cos x + (n+1) \int x^n \cos x dx. \quad (1)$$

Again, let $u = \sin x$ and $dv = x^n dx$,

$$\int x^{n+1} \cos x dx = x^{n+1} \sin x - (n+1) \int x^n \sin x dx. \quad (2)$$

Let $n = 0$ and substitute in (1) and (2),

$$\int x \sin x dx = -x \cos x - \sin x,$$

$$\int x \cos x dx = x \sin x - \cos x.$$

Let $n = 1$ and substitute in (1) and (2),

$$\int x^2 \sin x dx = -x^2 \cos x + 2 \int x \cos x dx$$

$$= -x^2 \cos x + 2x \sin x - 2 \cos x,$$

$$\int x^2 \cos x dx = x^2 \sin x - 2 \int x \sin x dx$$

$$= x^2 \sin x + 2x \cos x - 2 \sin x.$$

Let $n = 2$ and substitute in (1) and (2),

$$\int x^3 \sin x dx = -x^3 \cos x + 3 \int x^2 \cos x dx$$

$$= -x^3 \cos x + 3x^2 \sin x + 3.2x \cos x - 3.2 \sin x$$

$$\int x^3 \cos x dx = x^3 \sin x - 3 \int x^2 \sin x dx$$

$$= x^3 \sin x + 3x^2 \cos x - 3.2x \sin x - 3.2 \cos x.$$

Let $n = 3$ and substitute in (1) and (2), &c.

The laws of the series are thus determined, and may be expressed:

$$\begin{aligned} \int x^m \sin x dx &= -x^m \cos x + m x^{m-1} \sin x + m(m-1) x^{m-2} \cos x \\ &\quad - m(m-1)(m-2) x^{m-3} \sin x - \dots \end{aligned}$$

$$\begin{aligned} \int x^m \cos x dx &= x^m \sin x + m x^{m-1} \cos x - m(m-1) x^{m-2} \sin x \\ &\quad + m(m-1)(m-2) x^{m-3} \cos x + \dots \end{aligned}$$

[These integrals may be found at p. 265 of Hirsch's Integral Tables, but they were computed by Mr. Shack without suspecting that they were known forms.—Ed.]

NOTE ON THE SOLUTION OF PROB. 373.—We have received from Prof. H. T. Eddy, a brief and elegant solution of (373), in which several errors which occur in the published solution are pointed out and corrected. We have also received from Mr. Adcock the following corrections of his solution of that problem (see p. 55):

“I find two errors in the MS. of my solution of (373). In the printed solution on page 56, second line, strike-out the words, “and positive downwards”, and, in fourth line from bottom, for “ $dx \div ds = -y \div r$ ”, read $2\rho dx = ds^2$. From these two corrections there follow two resulting corrections, viz.; in lines 8 and 9, change the signs of the second member in the right-hand equation in each; and in line 3 from bot., for $-1 \div 2a$ read $-1 \div 2\rho$.

“*Errors in printing.* Strike-out g in line 10 and insert $+g$ in the 2nd parentheses in line 11; and in line 16 insert $+g$ immediately before the first bracket”.

SOLUTIONS OF PROBLEMS IN NUMBER TWO.

SOLUTIONS of problems in No. 2 have been received as follows:

From Prof. L. G. Barbour, 387, 390; Prof. W. P. Casey, 387, 388, 389, 391; Prof. A. B. Evans, 387; George Eastwood, 390; W. E. Heal, 387, 388, 389; Prof. A. Hall, 391; Prof. J. Scheffer, 387; Isaac H. Turrell, 390.

Prof. Casey should have been credited, in No. 2, for a solution of 383.

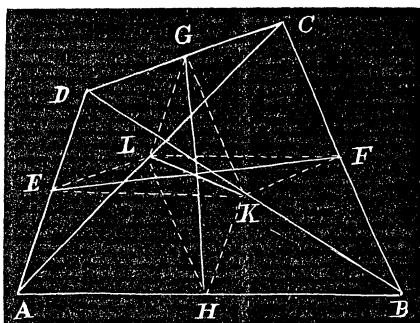
387. *By Prof. L. G. Barbour.*—“Given the length of each side of any quadrilateral, and the distance from the middle point of any side to that of the side opposite. Required the distance from the middle point of one of the other sides to that of the side opposite.”

SOLUTION BY W. E. HEAL.

Let EF , GH be the lines joining the middle points of opposite sides of the quadrilateral $ABCD$, and LK the line joining the middle points of the diagonals.

Put $AB = 2a$, $BC = 2b$, $CD = 2c$, $DA = 2d$, $EF = k$, $GH = h$.

Then $FL = EK = a$, $GH = HL = b$, $EL = FK = c$, $GL = HK = d$.



And, since $GHLK$ and $EFLK$ are parallelograms,

$$h^2 + (LK)^2 = 2(a^2 + c^2),$$

$$k^2 + (LK)^2 = 2(b^2 + d^2).$$

$$\therefore h^2 - k^2 = 2(a^2 - b^2 + c^2 - d^2).$$